

Wind Energy Math Calculations

Calculating the Tip Speed Ratio of Your Wind Turbine

The *Tip Speed Ratio (TSR)* is an extremely important factor in wind turbine design. TSR refers to the ratio between the wind speed and the speed of the tips of the wind turbine blades.

$$\text{TSR } (\lambda) = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}}$$

If the rotor of the wind turbine spins too slowly, most of the wind will pass straight through the gap between the blades, therefore giving it no power!

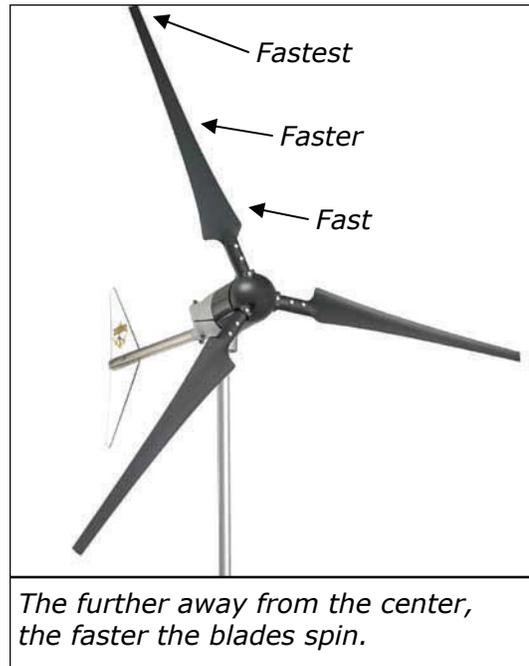
But if the rotor spins too fast, the blades will blur and act like a solid wall to the wind.

Also, rotor blades create turbulence as they spin through the air. If the next blade arrives too quickly, it will hit that turbulent air.

So, sometimes it is actually better to *slow down* your blades!

Wind turbines must be designed with optimal tip speed ratios to get the maximum amount of power from the wind.

Before we can calculate the tip speed ratio, we need to know how long it takes the rotor to make one full revolution.



Measuring Revolution Time (i.e. RPM)

It is easy to find the distance travelled by the blade tip, but finding how long it takes to go that distance can be tricky. There are several ways to find revolutions per minute (RPM), but there is not a single simple way!

- Manually—try to count how many times the blades revolve in a certain time period. It helps to mark or color one blade differently so you can be sure of when the turbine makes a full revolution.
- Use a real tachometer—These are great, but can be pricy. Try the “Hangar 9 Tachometer” or the “Extech Pocket Tachometer”
- If you’re crafty, you can use a bicycle computer. Attach the magnet to the rotor and the sensor to a non-moving part of the turbine. You can find detailed instructions here:

<http://www.reuk.co.uk/Use-a-Cycle-Computer-to-Measure-Turbine-RPM.htm>

You need to know how many seconds it takes the rotor to spin around *one* time. If you found RPM, you will need to convert this number. There are 60 seconds in one minute, so just divide 60 by your RPM value. That will tell you how many seconds it takes to make one revolution.

How To Find The Tip Speed:

1. Measure the rotor radius (length of one blade)
2. Speed = distance divided by time. The distance travelled is the circumference ($2\pi r$).
3. Speed:
$$V = \frac{2\pi r}{T \text{ (time)}}$$

The blades travel one circumference ($2\pi r$) in a rotation time of T (seconds).

Now you see why we need to know how long it takes to make one full revolution!

How To Find The Wind Speed:

The only exact way to know wind speed is to use a meter. The Kestrel 1000 wind speed meter is a great tool. But, if you don't have one, here's a quick guide:

Regular Box Fan on HI (3) Setting:

<u>Distance</u>	<u>Wind Speed (m/s)</u>
1 Foot	4–5
2 Feet	2.5–3.5
3 Feet	1.5–2

Regular Box Fan on MED (2) Setting:

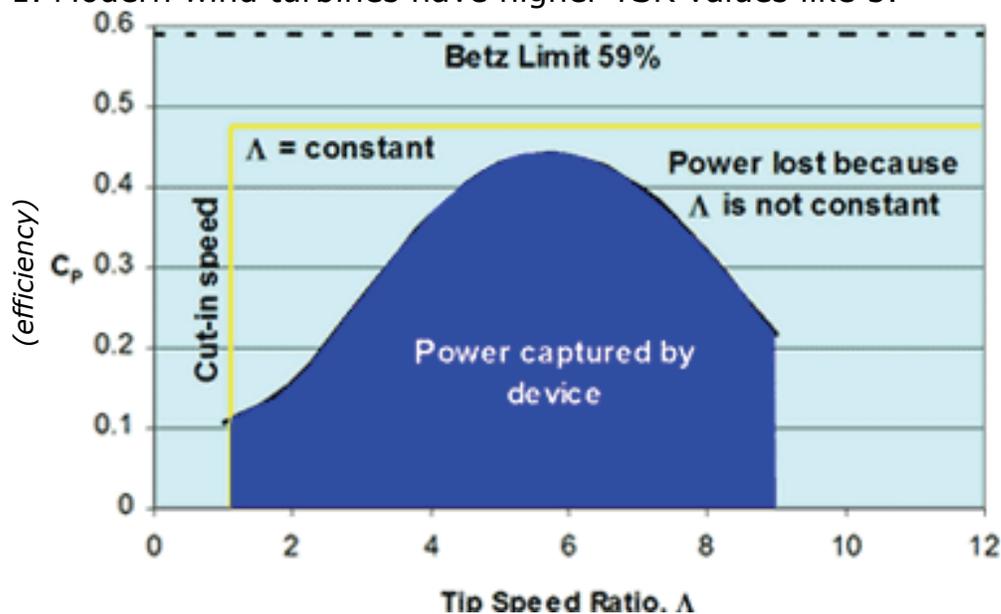
<u>Distance</u>	<u>Wind Speed (m/s)</u>
1 Foot	3–3.5
2 Feet	2–2.5
3 Feet	1–2

You need to make sure that your units are all the same. For example: You can't use a wind speed in Miles/Hour and a tip speed in Meters/Second. This formula will help if you need to convert units of velocity:

1 meter per second equals 2.237 miles per hour.

Now that you've made all your measurements, you should be able to figure out your tip speed ratio (TSR). You already calculated the tip speed in the equation described above (distance travelled divided by time taken for one revolution). Now take that number and divide it by your wind speed!

If your TSR is above 1, that means there is *lift* involved to make your blades spin faster than the wind speed. If your TSR is below 1, there is a lot of drag going on! Old windmills used to lift weights, grind grain, or pump water probably had TSRs around 1. Modern wind turbines have higher TSR values like 5.



Why is This Important???

Knowing the tip speed ratio of your turbine will help you maximize the power output and efficiency of your wind turbine. Remember that if your rotor spins too slowly, a lot of wind will pass through the gaps between the blades rather than giving energy to your turbine. But if your blades spin too quickly, they could create too much turbulent air or act as a solid wall against the wind. So, if you want to maximize your turbine's efficiency, you've got to calculate the perfect Tip Speed Ratio.

But that raises the question:

HOW DO YOU KNOW THE PERFECT TIP SPEED RATIO???

If you want the optimum Tip Speed Ratio for maximum power output, this formula has been empirically proven:

$$\lambda \text{ (max power)} = \frac{4\pi}{n} \quad (n = \text{number of blades})$$

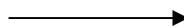
Of course, there is always a cheat sheet if you're feeling lazy:

# of Blades		Optimum TSR
2	—	Around 6
3	—	Around 4—5
4	—	Around 3
6	—	Around 2



Wind Turbine Tip Speed Visualization using smoke at the National Renewable Energy Laboratory (NREL).

Did you notice that the blades on this machine are down-wind of the tower?



Sample Tip Speed Ratio Measurements



This data was collected using a Kidwind Geared PVC Turbine and a Honeywell HF 810 Fan.

The PVC Geared Turbine was placed 30 inches from the fan and the hub of the wind turbine was matched to the center of the fan.

The blades were 8" long and approximately 1.5" wide.

Blade Size: 8 inches

Diameter: 17 inches

Fan Speed	Measured Wind Speed (m/s)	Measured Wind Speed (mph)	Blade RPM	Tip Speed Ratio	Generator RPM (6.1:1 Gearbox)	Voltage
Fast	4.4	9.8	750	3.9	4575	3.85
Medium	3.8	8.5	520	3.1	3172	3.1
Slow	2.9	6.5	380	3.0	2318	2.1

RPM was measured using a Horizon Hobby Digital Voltmeter/Tachometer (DVT).

Voltage was measured without placing a load on the turbine. If a load (bulb, pump, motor) is placed on the turbine you should expect lower RPM & voltage.

What is Tip Speed Ratio? That is the ratio of the blade tips to the wind speed. In this spreadsheet TSR was calculated:

These values are only good for this set of blades and this fan. You can download this spreadsheet at www.kidwind.org and enter your own values to calculate TSR. You will need a device to measure wind speed and blade RPM.



Determining Tip Speed: (Circumference = 53.4 inches)

Fan Setting	RPM	Inches/min	Feet/min	Mile/min	Mph
Fast	750	450055.3	3337.9	0.632	37.93
Medium	520	27771.7	2314.3	0.438	26.30
Slow	380	20294.7	1691.2	0.320	19.22
Theoretical	1500	80110.6	6675.9	1.264	75.86

Sample Problems:

1. If you have a wind turbine with three blades, each 4 meters long, what distance does the tip of each blade travel in one full revolution?
2. If this turbine is rotating at a rate of 42 Revolutions per Minute (RPM), how long does it take to make one full revolution?
3. Based on your answers from 1 and 2, calculate how fast the tips of this wind turbine are moving through the air.
4. If the wind is blowing at 6 meters per second, what is the tip speed ratio of this turbine? Use your answers from questions 1-3 to help solve this problem.
5. According to the "optimal" tip speed ratio for this three-bladed turbine, are these blades moving too fast or too slow?
6. An offshore wind turbine with three 60 meter blades rotates at a leisurely 12 RPM. The wind is whipping along at 18 meters per second. What is the tip speed ratio for this turbine? How does this compare to the "optimal" tip speed ratio for this turbine?
7. You're Kidwind turbine has blades that are 0.25 meters long. They are spinning rapidly at 600 RPM. What is the tip speed?
8. You have measured the wind speed to be 9 **miles per hour**. How fast is the wind blowing in meters per second?
9. Using the data from questions 7 and 8, calculate the Tip Speed Ratio of this wind turbine.

Did You Know?

Without the use of the gearbox, utility scale wind turbines using an ordinary generator would have to rotate at over 1,000 rpm to generate the 60 Hz AC electricity we use in our homes. That means the tips of those long blades would be rotating at over *TWICE THE SPEED OF SOUND!!!*

Good thing somebody invented gearboxes...