Kidwind Math Sample Problems — Answer Key

Swept Area Questions:

1. The blades are 45 meters long. The length of one blade is equal to the radius. $\Pi \times 45^2$ = **6362 meters squared.**
2. The diameter is 60 meters, so the radius is 30 meters. $\Pi \times 30^2$ = **2827 meters squared.**
3. $P = 1/2 \times 1.225\ kg/m^3 \times 6362m^2 \times 10^3$ = 9,546,976.25 Watts or about 9.55 Megawatts
4. $P = 1/2 \times 1.225\ kg/m^3 \times 6362m^2 \times 12^3$ = 16,497,174.96 Watts or about 16.5 Megawatts.
5. $P = 1/2 \times 1.225\ kg/m^3 \times 2827m^2 \times 20^3$ = 13,852,300 Watts or about 13.85 Megawatts.
6. There are many losses in drag and friction. The Betz Limit states that the best theoretical wind turbine can still only be 59% efficient.

Tip Speed Ratio Questions:

1. The length of one blade is equal to the radius. The distance travelled is equal to the circumference of the circle created by the blades. Circumference of a circle = $2\Pi r$. So, $(2 \times \Pi \times 4)$ = **25.13 meters**.

2. $$\frac{\text{1 Minute}}{\text{42 Revolutions}} \times \frac{\text{60 Seconds}}{\text{1 Minute}} = \frac{60}{42} =$$ **1.43 Seconds** per revolution

3. Velocity = distance divided by time. $\frac{\text{25.13 meters}}{\text{1.43 seconds}}$ = **17.6** m/s

4. $\text{TSR} = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}} = \frac{17.6}{6}$ = **2.93**

5. Optimal TSR = $\frac{4\Pi}{n} = \frac{4\Pi}{3}$ = 4.2—The blades are moving too slow

6. $2 \times \Pi \times 60$ = 377 meters travelled. $\frac{60}{12}$ = 5 seconds per revolution.
$\frac{\text{377 meters}}{\text{5 seconds}}$ = 75.4 m/s. TSR = 75.4/18 = 4.2. This is the optimal TSR for this wind turbine.

Tip Speed Ratio Questions (continued):

7. 2 x ∏ x 0.25 = 1.57 meters travelled. 600 rpm = 10 revolutions per second. 1 revolution = 1.57 meters. **Tip speed = 15.7** meters per second.

8. 1 meter per second = 2.237 miles per hour. 9/2.237 = 4.02 meters per second.

9. $\text{TSR} = \dfrac{\text{Tip Speed of Blade}}{\text{Wind Speed}} = \dfrac{15.7}{4.02} = \mathbf{3.91}$

<u>Calculating Height Questions</u>

1. 3/1.7 = x/164.333   x = **290 feet**
2. Answers vary… H/8 = x/500 (H=student's height). So, if they are 5 feet tall, 5/8 = x/500   x = 312.5 feet
3. With method 2, the vertical height of the tower is equal to the horizontal distance from where you stood to the base of the tower plus your height. 2.2 x 111 = 244.2 + 5.75 = 249.95 or about **250 feet**.
4. 7 x 32.5 = **227.5 feet**.
5. Answers vary.

<u>Gear Ratios Questions</u>

1. 1680/12 = 140   The gear ratio is **140:1**
2. 2100/120 = **17.5 RPM**
3. 600 x 6 = **3600 RPM**   3600/790 = **4.56 Volts**
4. Final gear ratio = (1260/70) x (70/14) = **90:1**. 90 x 15 = **1350 RPM**

<u>Word Problems</u>

1. $2,100,000/14,000,000 = **$0.15 per kWh**
2. $2,100,000 x 10 = **$21,000,000**
3. $15.750,000/$2,100,000 = **7.5 years**
4. 14,000,000 x 0.9 = 12,600,000… 2,100,000/12,600,000 = **$0.1667** or about 17 cents per kWh
5. 14,000,000 x 0.9 = 12,600,000… 12,600,000 x $0.15 = $1,890,000 per year. $15,750,000/$1,890,000 = **8.333 years** (8 1/3).
6. 10 - 8.333 = 1.667… 1.667 x $1,890,000 = **$3,150,000**
7. 2.1 x 0.6 x 42 = **59.92 MW average**
8. 59.92 x 24 x 365 = **463,579.2 MWh per year**. 463,579.2 MWh x 1,000 kW/MW = **463,579,200 kWh per year**
9. 463,579,200 kWh x $0.15 = **$69,536,880 per year**
10. $69,536,880 x 0.05 = **$4,172,212.80 per year**

Coefficient of Power Problems:

1. 0.62 x 0.59 = 0.37 or about 37%
2. 50 volts x 20 amperes = 1,000 watts. 1,000/3,900 = 0.26 or 26%
3. a. $P = 1/2 \times 1.23 \times 2\Pi(1) \times 12^3 = 6{,}677.3$ Watts.
   b. 12 volts x 33 amps = 393 Watts
   c. Cp = 393/6,677.3 = 0.06 or about 6%.

4. This man is probably a con artist! The fact that he is claiming that his turbine is 73% efficient is how you know. This is impossible since the Betz limit is 59%. Therefore, no wind turbine can exceed this efficiency! Also, it is a really bad idea to put a wind turbine on your roof. This can cause structural damage to your house and also create a good bit of noise due to vibrations. Tell the salesman to go away!

5. 25 is not a good tip speed ratio. Once again, this man is a con-artist! When the blades are spinning too fast, they appear like a solid wall against the wind. That reduces efficiency. Also, each blade creates turbulent air in its wake as it spins around. If the next blade arrives too soon, it will hit that turbulent air, which also reduces efficiency.